A discrete Search and Rescue Problem under uncertain interval parameters

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Keywords: Intervals, Search And Rescue, WTA, Uncertainty

Introduction

The Search And Rescue Problem (SARP) can be formulated in an environment subject to both an objective uncertainty (randomness inherent to nature) and **subjective uncertainty** (lack of knowledge about the state of the world). This second type of uncertainty, concerning problem parameters, can be represented using intervals [1].

In this work, SARP is formulated as the minimization of sum $[F] \in \mathbb{IR}$ of injury level intervals $[d_j] \in \mathbb{IR}$ over n heterogeneous wounded individuals j, each associated with a value (e.g., injury severity) $[v_j] \in \mathbb{IR}$, after assigning m caregivers i with heterogeneous skills. Each caregiver-to-wounded assignment (i, j) is characterized by distinct, independent, and **cumulative** probability $[p_{ij}] \subset [0, 1]$ that caregiver i successfully treats individual j. A caregiver is assigned to a wounded individual *iff* $x_{ij} = 1$. The goal is then to minimize:

$$[F] = \sum_{j=1}^{n} [v_j] \prod_{i=1}^{m} (1 - [p_{ij}])^{x_{ij}} \qquad (1)$$

Subject to the following constraints (with **binary** decision variables (2): at most one wounded individual per caregiver (3); a minimum required care efficiency smin_j for each assignation (4); a maximum injury threshold after all assignations bmax_j (5)):

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in \{1,m\} \times \{1,n\} \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} \leq 1 \quad \forall i \in \{1,m\} \tag{3}$$

$$x_{ij} = 1 \implies [v_j] \times [p_{ij}] \subset [\text{smin}_j, 1.0] \quad \forall (i,j) \in \{1,m\} \times \{1,n\} \tag{4}$$

$$[v_j] \prod_{i=1}^{m} (1 - [p_{ij}])^{x_{ij}} \subset [0.0, \text{bmax}_j] \quad \forall j \in \{1,n\} \tag{5}$$

The use of intervals allows encoding uncertainty at the input level, with parameters defined over $\mathbb{IR}$: $v_j \in [\underline{v_j}; \overline{v_j}]$, $p_{ij} \in [\underline{p_{ij}}, \overline{p_{ij}}]$. Interval arithmetic is also used to verify constraints (4) and (5) to propagate uncertainty through to the output objective $F \in [\underline{F}, \overline{F}]$.

This multiplicative (non-linear) formulation has as main interest the accumulation of probability $[p_{ij}]$. It can be seen as an extrapolation of the Weapon Target Assignment Problem (WTAP [2]) used in military applications.

Related Work and Motivation

To date according to the state of the art, existing solvers do not address binary-interval problems of this type. A single-objective, balanced assignment problem (linear, one-to-one) with uncertain interval parameters is solved based on interval midpoints [3], which is extended to unbalanced problems [4]. A similar but multi-objective problem is solved through variable substitution, by moving binary variables from the objective function to the constraints [6]. Also, a bi-objective problem is solved by disjoint enumeration across objectives and parameter bounds [7].

Contribution

We address a SARP, using a Branch and Bound method implemented in C++, with the following contributions:

- **Interval parameters** in a WTA-dual problem: unbalanced (arbitrary number of caregivers and wounded individuals), **many-to-one** assignment, **multiplicative cumulative** objective function.
- Direct manipulation of intervals during computation (using the **IBEX** library [9]) rather than indirect approximation methods, and so an output representation of reduced injuries as an interval.

Figure 1 presents an example output of the algorithm. The uncertainties in resulting injuries (interval widths) are computed using interval arithmetic based on input uncertainties. As expected, the **reduced injury intervals**, shown as blue segments, are all upper-bounded by the maximum injury constraints (5) (red dots), i.e. all individuals are sufficiently treated. The wounded individual 2 is deliberately left untreated without unsatisfying the constraints (5). Notably, the algorithm handles negative output values, as shown by wounded individual 6: caregiver attention might partially be better allocated elsewhere.

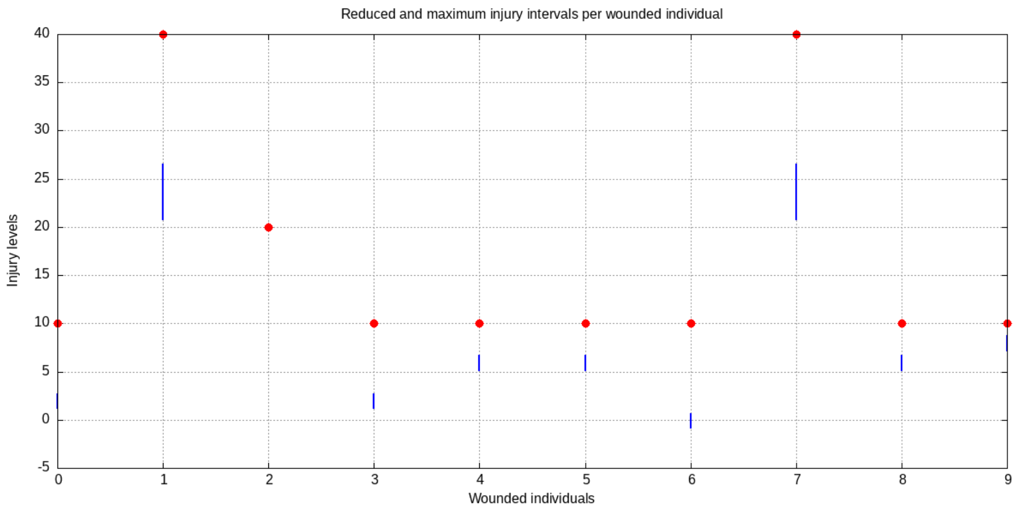

Figure 1: Intervals of reduced injuries

Acknowledgement

This work is funded by KNDS, a Franco-German company of the defense industry. This project also received funding from the French National Research Agency (ANR).

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