Inner and outer approximation of the workspace of a robotic arm

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Keywords: Robotics, Workspace, Parallelepipeds

Introduction

The objective of this presentation is to compute the workspace of a 2D robotic arm as depicted Figure 1. The presented method relies first on parallelepipeds to compute the boundary of the workspace. These parallelepipeds are a compromise between the simplicty of boxes and the low pessimism of zonotopes [3].

To be able to describe the whole boundary of the workspace, a study of the singularities of the robotic arm will be carried out. Indeed this kind of arm has a singularity when the two segments are aligned, and it needs to be handled apart.

From the resulting enclosure of the boundary a Delaunay triangulation [2] [1] will be used to get a first inner approximation of the workspace. This will allow us to remove the fake boundary, i.e. the parallelepipeds which should be classified as part of the inner approximation. Finally an inner and an outer approximation of the workspace will be obtained.

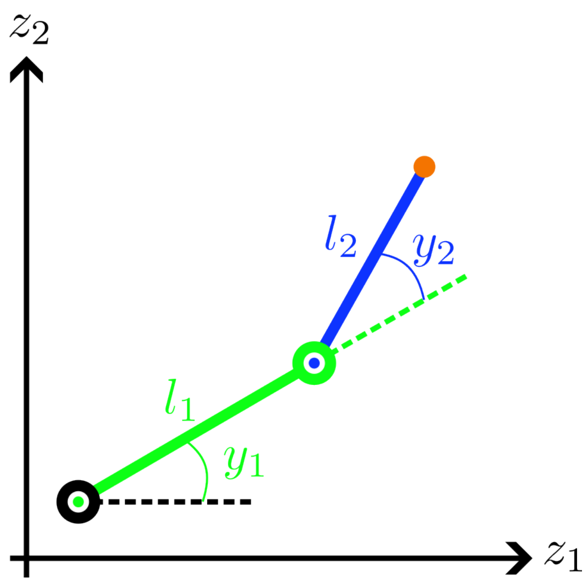

Figure 1: 2D robotic arm

Main results

Let us consider the robotic arm defined Figure 1.

The position of the effector (z_1, z_2) is defined by:

$$\mathbf{f}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} l_1\cos(y_1) + l_2\cos(y_1+y_2) \\ l_1\sin(y_1) + l_2\sin(y_1+y_2) \end{pmatrix}$$

For numerical applications we will take $l_1 = l_2 = 0.3$ and bound the angles to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Figure 2 shows the boundary of the workspace covered by parallelepipeds.

From this Figure 3 shows in green the inner approximation, in yellow the boundary (made out of parallelepipeds) and in red the outside of the workspace. To get this this result each set was first decompose into convex subsets before triangulating each subset. Note that an outer approximation is given by the union of the inner approximation and the boundary.

Finally we are able to remove fake boundaries, i.e. remove the parallelepipeds which can be classified as inside the workspace. We

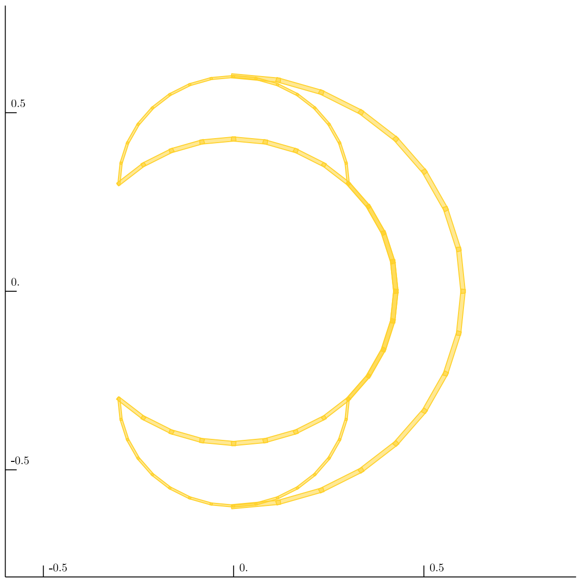

Figure 2: Boundary of the workspace

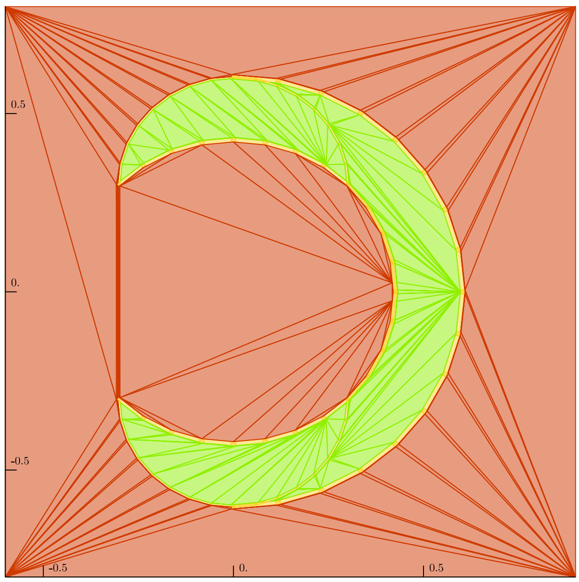

Figure 3: Initial Inner (green) and outer (green+yellow) approximation of the workspace

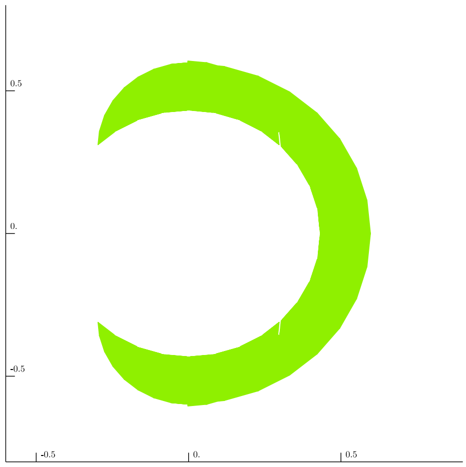

Figure 4: Inner Approximation

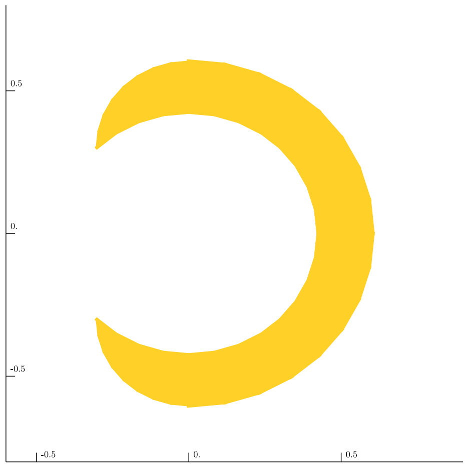

Figure 5: Outer Approximation

Figure 6: Workspace of the 2D robotic arm

then get the result of Figure 6 for the inner and outer approximation of the workspace.

Acknowledgement

Thanks to the french Defence Innovation Agency (AID) and the Brittany Region.

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