Ultra-wideband Sensor Based Robust Model Predictive Control

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Introduction

We propose a robust nonlinear model predictive controller (NMPC) leveraging ultra-wideband (UWB) positioning for autonomous control of a power wheelchair (PW). Our goal is to drive the PW to a target pose. NMPC is well-suited for PW control, as it naturally handles actuator limitations and user comfort constraints. At each UWB measurement epoch, an optimal control problem is solved to generate a control sequence, of which only the first value is applied to the PW. The system evolution is predicted over a receding horizon using a nonlinear model.

An indoor UWB positioning system typically consists of fixed anchors and mobile tags. Distances are measured using two-way ranging, which calculates the time of flight of signals between tags and anchors. These measurements are influenced by various factors, including signal noise, non-line-of-sight (NLOS) conditions, multipath propagation, inherent biases in radio modules, and inaccuracies in anchor positioning. Combined with model uncertainties, these errors can lead to inaccurate pose estimates and potential constraint violations, such as collisions. To ensure robust constraint satisfaction, we employ set-membership state estimation, which computes an outer approximation of all states consistent with UWB measurements and propagates this set through uncertain dynamics over the receding horizon. The control sequence is then optimized to guarantee that all possible trajectories remain within the constraints.

An algorithm solving the NMPC problem in a validated way, using interval analysis, has been proposed in [1]. A similar approach is the tube-based MPC [2], [3], in which an ancillary feedback controller is used to ensure that the actual state remains within an invariant tube around the trajectory computed by solving the nominal NMPC problem. To guarantee constraint satisfaction for all possible poses within the tube, state and input constraints are appropriately tightened.

In this work, we propose a robust NMPC framework that avoids the offline computation of both the ancillary controller and the associated invariant set, which is a complex task for general nonlinear systems [4].

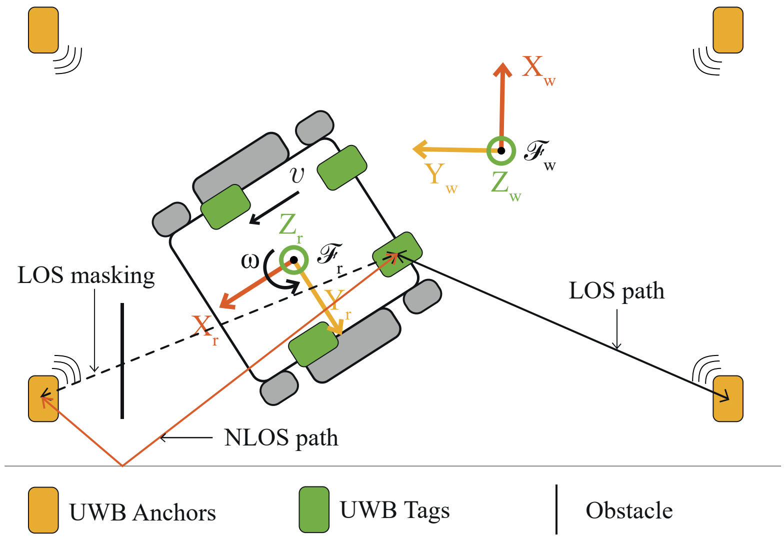

Figure 1: System Overview. Each UWB tag (green) measures its distance to each anchor (yellow). Range measurements can be affected by multipath or NLOS propagation.

System

The PW pose is denoted $\mathbf{x} = (x, y, \theta)^{\mathbf{T}}$, where (x, y) are the coordinate of the PW's center of rotation, and θ is the heading angle. The two control inputs $\mathbf{u}$ related to the PW are its linear velocity v and its angular velocity ω. The nonlinear discrete-time system of the PW is

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) \quad (1)$$
$$\mathbf{y}_k = \mathbf{g}(\mathbf{x}_k, \mathbf{v}_k) \quad (2)$$

where $\mathbf{f}$ is the system model and $\mathbf{g}$ is the observation model, and $\mathbf{v}_k$ is the measurement disturbance.

Robust NMPC

At each epoch k, UWB range measurements are used to estimate the PW pose. Assuming unknown but bounded measurement uncertainties, the set of all PW poses consistent with the UWB measurements can be conservatively over-approximated using set representations $\hat{\mathcal{X}}_{y_k}$. From the previous time step $k-1$, we have the set of all PW poses that are consistent both with the UWB measurements recorded at epoch $k-1$, and with the propagated sets of feasible poses from earlier epochs $\hat{\mathcal{X}}_{k-1}$. The optimized control sequence is denoted $\mathbf{u}^{\star}_{k-1} = \{\mathbf{u}^{\star}_{0|k-1}, \mathbf{u}^{\star}_{1|k-1}, \ldots, \mathbf{u}^{\star}_{N-1|k-1}\}$, with $\cdot_{i|k-1}$ the i^{th} control inputs from sampling time $k-1$. The set $\hat{\mathcal{X}}_{k-1}$ is propagated forward using model (1) while accounting for bounded uncertainties in the applied control inputs, resulting in a predicted set $\hat{\mathcal{X}}_{k|k-1}$. In the following, this is done using the natural interval extension for box representation. The natural inclusion function is denoted $[\mathbf{f}]$. The set of all possible states at epoch k is given by $\hat{\mathcal{X}}_k = \hat{\mathcal{X}}_{y_k} \cap \hat{\mathcal{X}}_{k|k-1}$.

Let $\mathcal{X}_{safe}$ denotes the set of feasible states and $\mathcal{U}$ the set of admissible control inputs. The uncertainty set $\hat{\mathcal{X}}_k$ is forward propagated to tighten the NMPC states constraints. As a result, the PW trajectory is guaranteed to satisfy state constraints, despite the presence of pose estimation uncertainty. The NMPC problem is formulated such as

$$\begin{aligned}
\{\mathbf{x}_k^\star, \mathbf{u}_k^\star\} = \underset{\{\mathbf{x}_{i|k}, \mathbf{u}_{i|k}\}}{\arg\min} \quad & \sum_{i=0}^{N-1} \ell(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}) + V_f(\mathbf{x}_{N|k}) \\
\text{s.t.} \quad & \mathbf{x}_{0|k} = \hat{\mathbf{x}}_k \\
& \mathcal{X}_{0|k} = \hat{\mathcal{X}}_k \\
& \mathbf{x}_{i+1|k} = \mathbf{f}(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}), \quad i = 0, \ldots, N-1 \\
& \mathcal{X}_{i+1|k} = [\mathbf{f}](\mathcal{X}_{i|k}, \mathbf{u}_{i|k}) \quad i = 0, \ldots, N-1 \\
& \mathbf{x}_{i|k} \in \mathcal{X}_{safe} \ominus \mathcal{E}_{i|k}, \quad i = 1, \ldots, N \\
& \mathbf{u}_{i|k} \in \mathcal{U}, \quad i = 0, \ldots, N-1
\end{aligned}$$

where $\ell(\cdot)$ is the stage cost, $\ominus$ is the Pontryagin set difference. The set $\mathcal{E}_{i|k} = \mathcal{X}_{i|k} \ominus \mathbf{x}_{i|k}$ is the possible deviation between the nominal state and the actual PW pose at prediction step i, from sampling time k. $V_f(\cdot)$ is the terminal cost.

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